A catalog of 3-regular matchstick graphs of girth 5 consisting of 54 – 68 vertices

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March 14, 2019

Abstract

This article exhibits the currently smallest known examples of 3-regular matchstick graphs of girth 5 consisting of less than 70 vertices.

1. Introduction

A 3-regular matchstick graph is a planar unit-distance graph whose vertices have all degree 3. The girth of a graph is the length of a shortest cycle contained in the graph. Therefore, no rigid matchstick graph of girth ≥ 4 exists, because such graphs contain only flexible subgraphs.

In 2010 Sascha Kurz and Giuseppe Mazzuoccolo proved that a 3-regular matchstick graph of girth 5 consists at least of 30 vertices and gave an example consisting of 180 vertices [1].

This article exhibits the currently smallest known examples of 3-regular matchstick graphs of girth 5 consisting of 54 – 68 vertices (see Table 1). The graphs were first presented by the author between January 24 – February 8, 2019 in a graph theory internet forum [4]. Each graph and its proof by construction can also be viewed on the authors website *mikematics.de*[1] with a software called Matchstick Graphs Calculator (MGC) [2]. This software runs directly in web browsers.[2] The MGC includes an animation function for representing the movement we are mentioning in our proof. It remains an open question whether such graphs with less than 54 vertices exist.

In the PDF version of this article the vector graphics can be viewed with the highest zoom factor to see the smallest details.

vertices	54	58	60	64	66	68
examples	1	1	1	2	3	3

Table 1: Number of known examples of 3-regular matchstick graphs of girth 5 with less than 70 vertices.

[1] http://mikematics.de/matchstick-graphs-calculator.htm
[2] For optimal functionality and design please use the Firefox web browser.

2. Currently known examples of 3-regular matchstick graphs of girth 5 consisting of 54 – 68 vertices

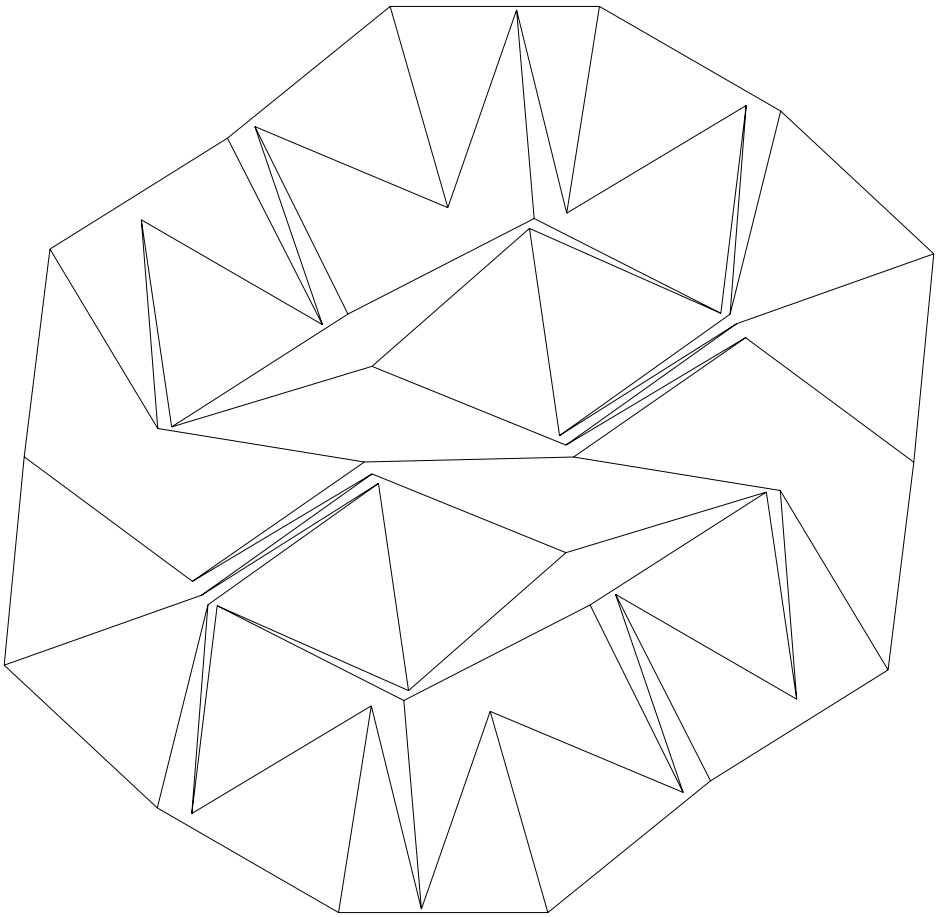

Figure 1: 54 vertices, point symmetry

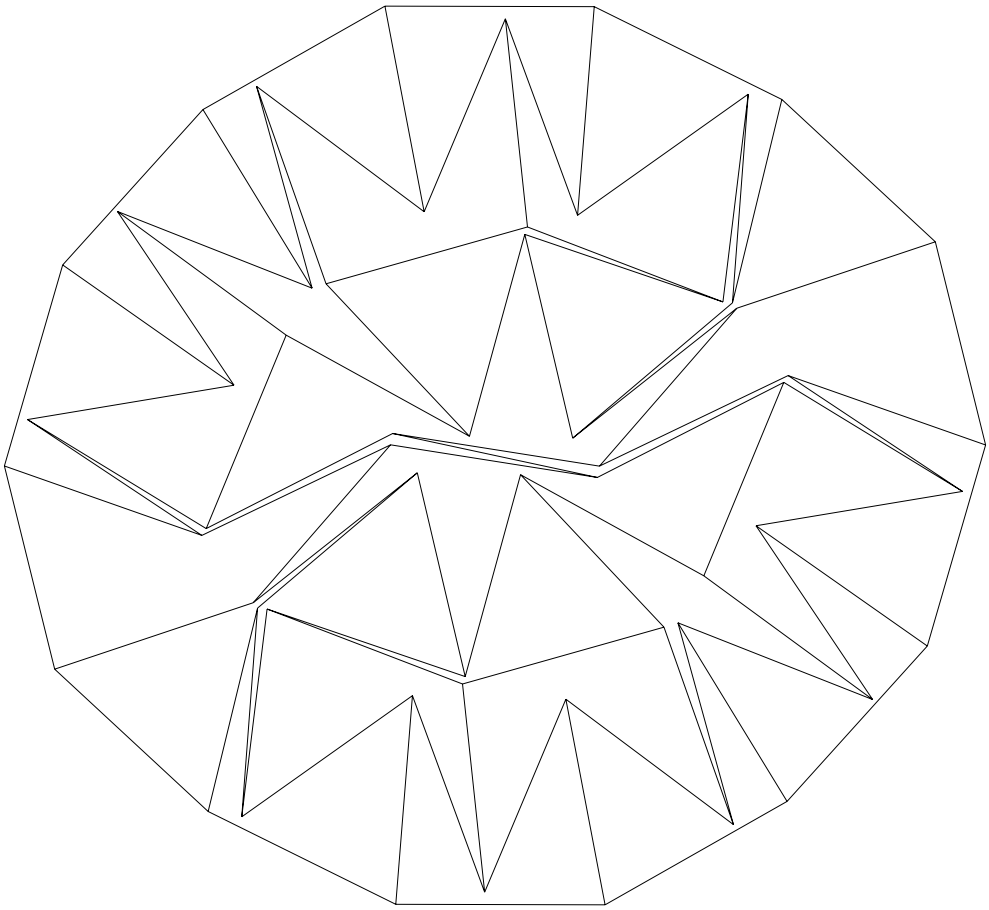

Figure 2: 58 vertices, point symmetry

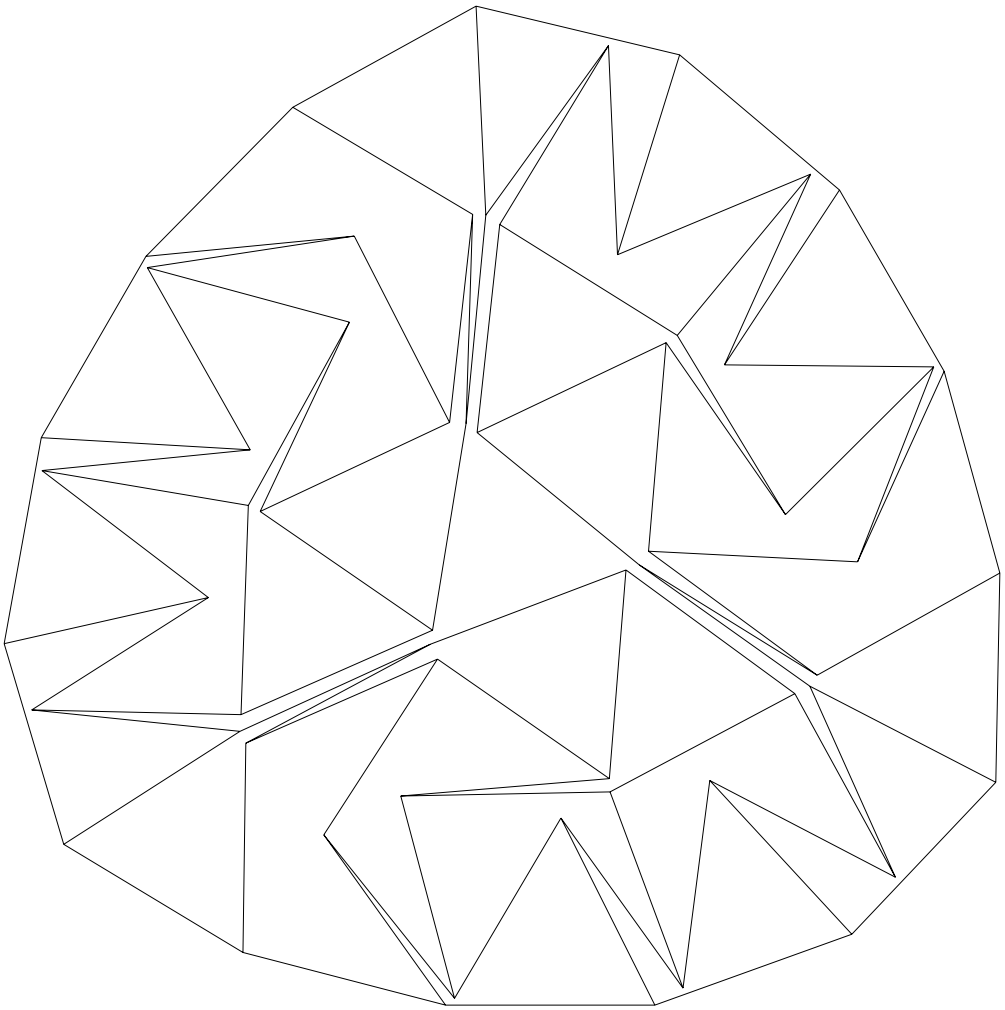

Figure 3: 60 vertices, rotational symmetry of order 3

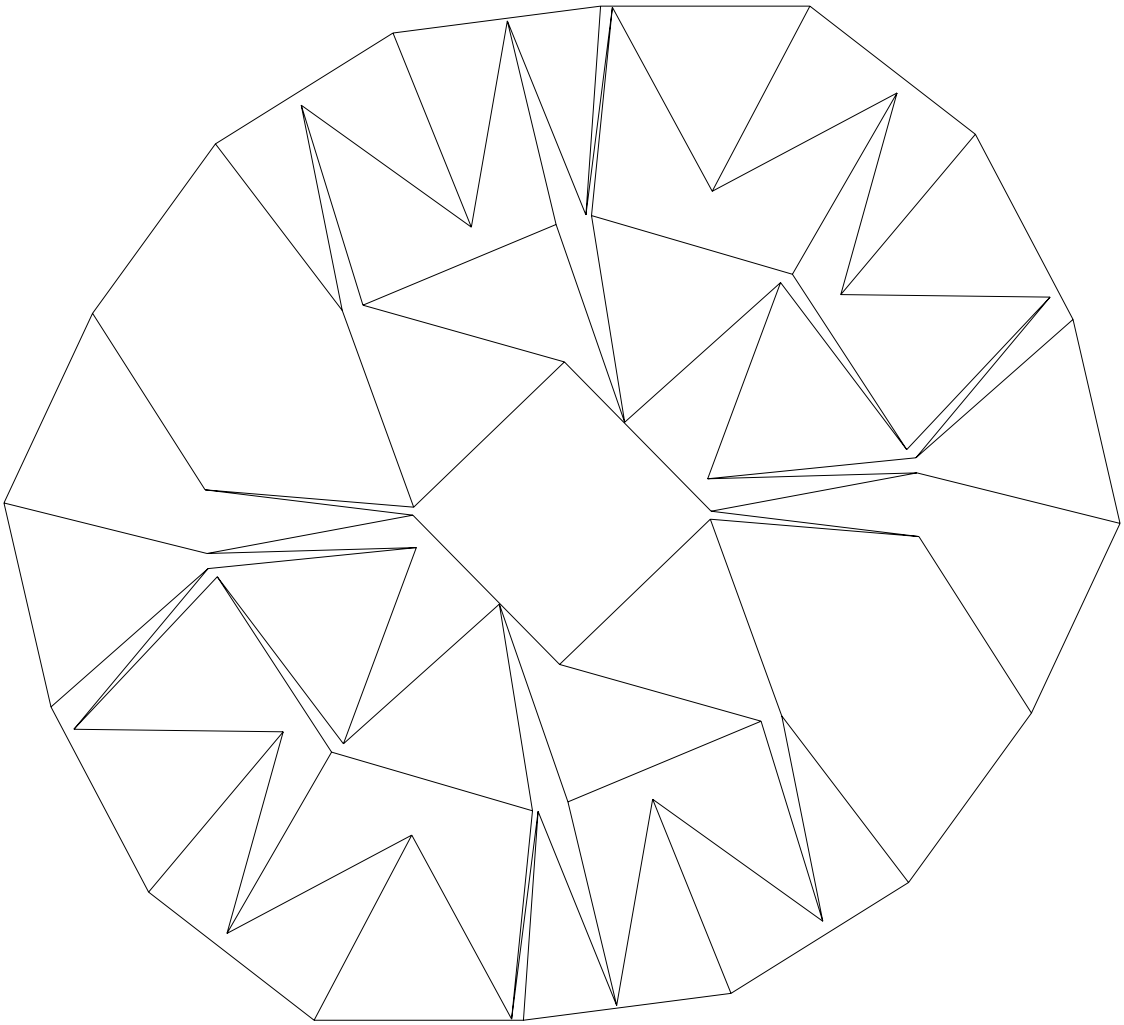

Figure 4: 64 vertices, point symmetry

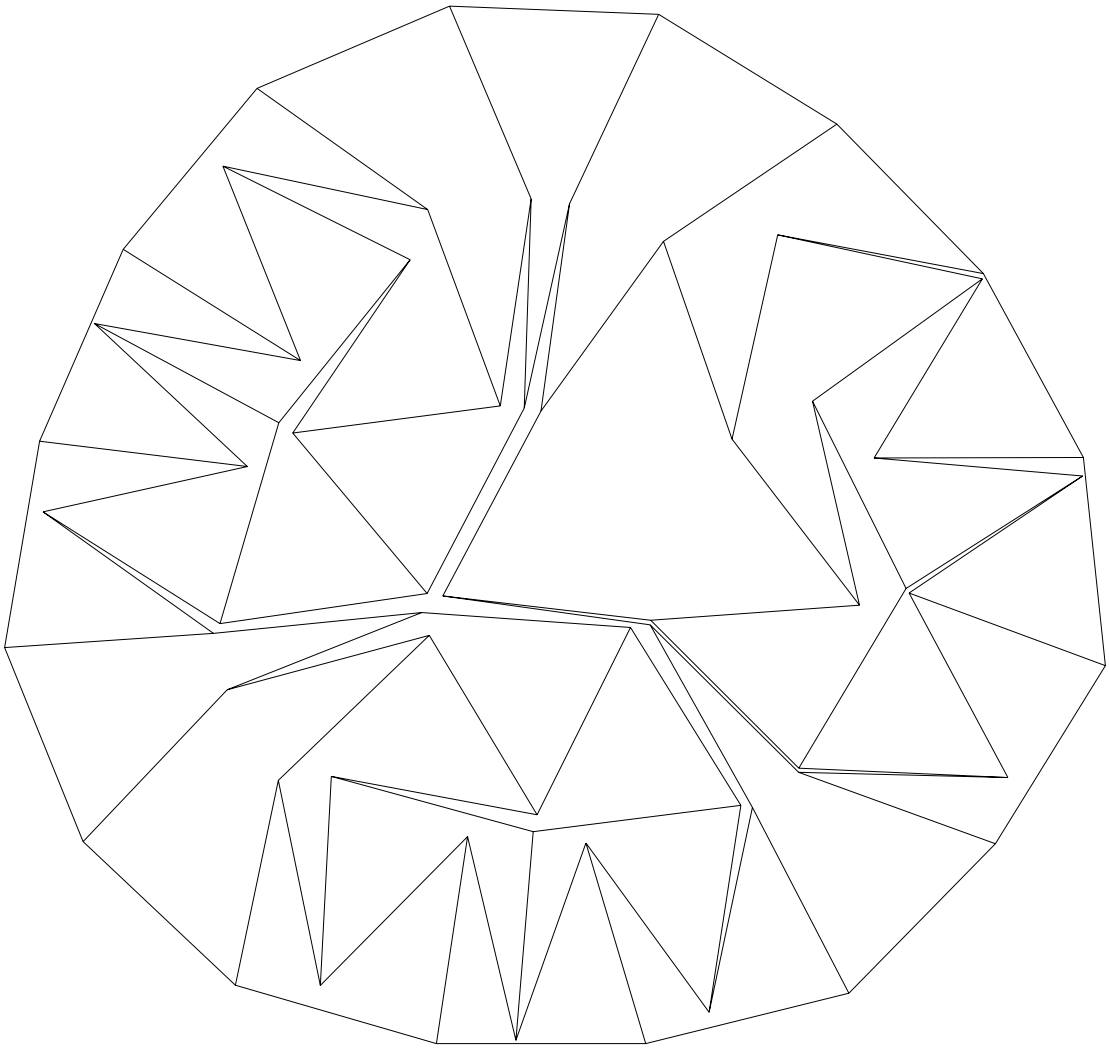

Figure 5: 64 vertices, asymmetric

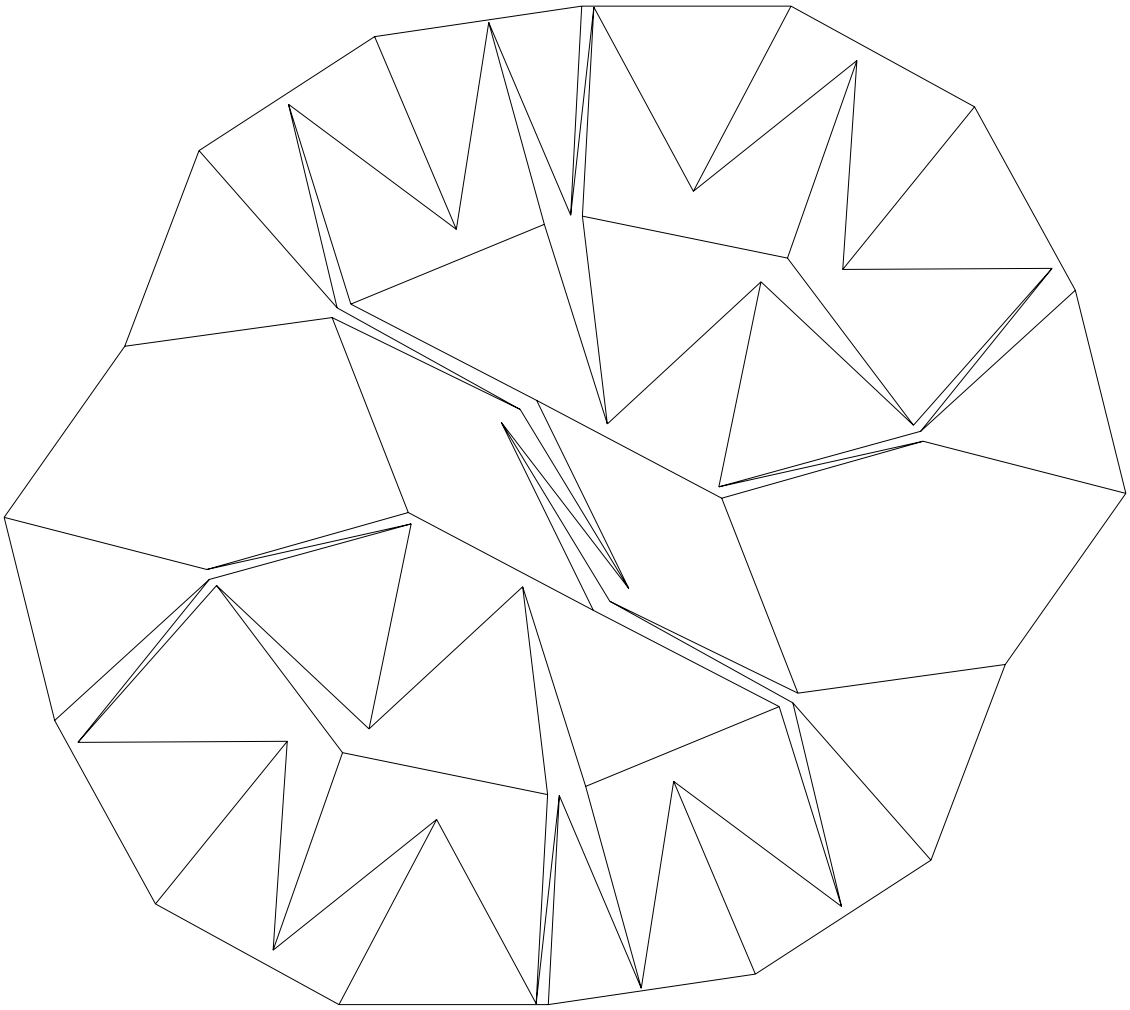

Figure 6: 66 vertices, point symmetry

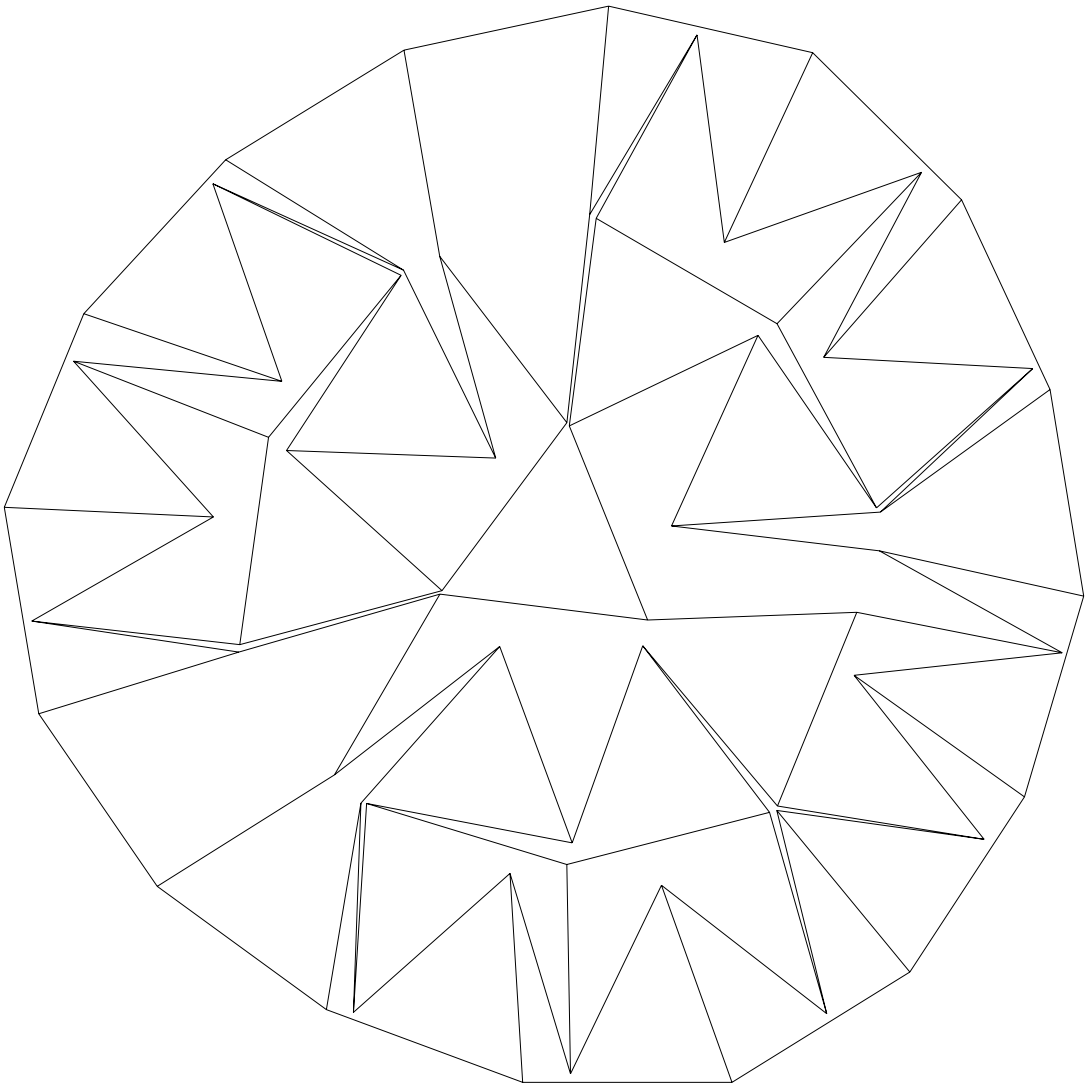

Figure 7: 66 vertices, asymmetric

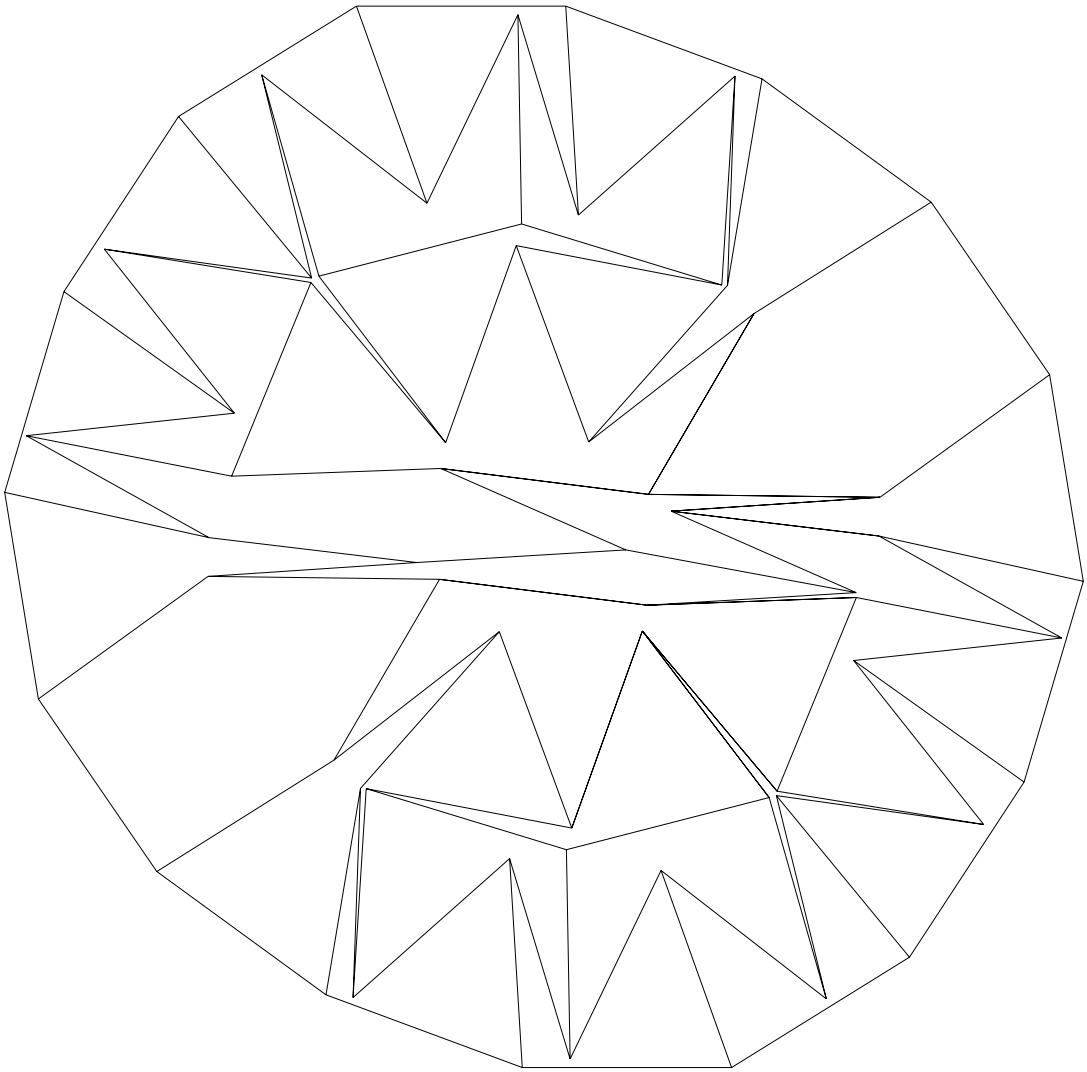

Figure 8: 66 vertices, asymmetric

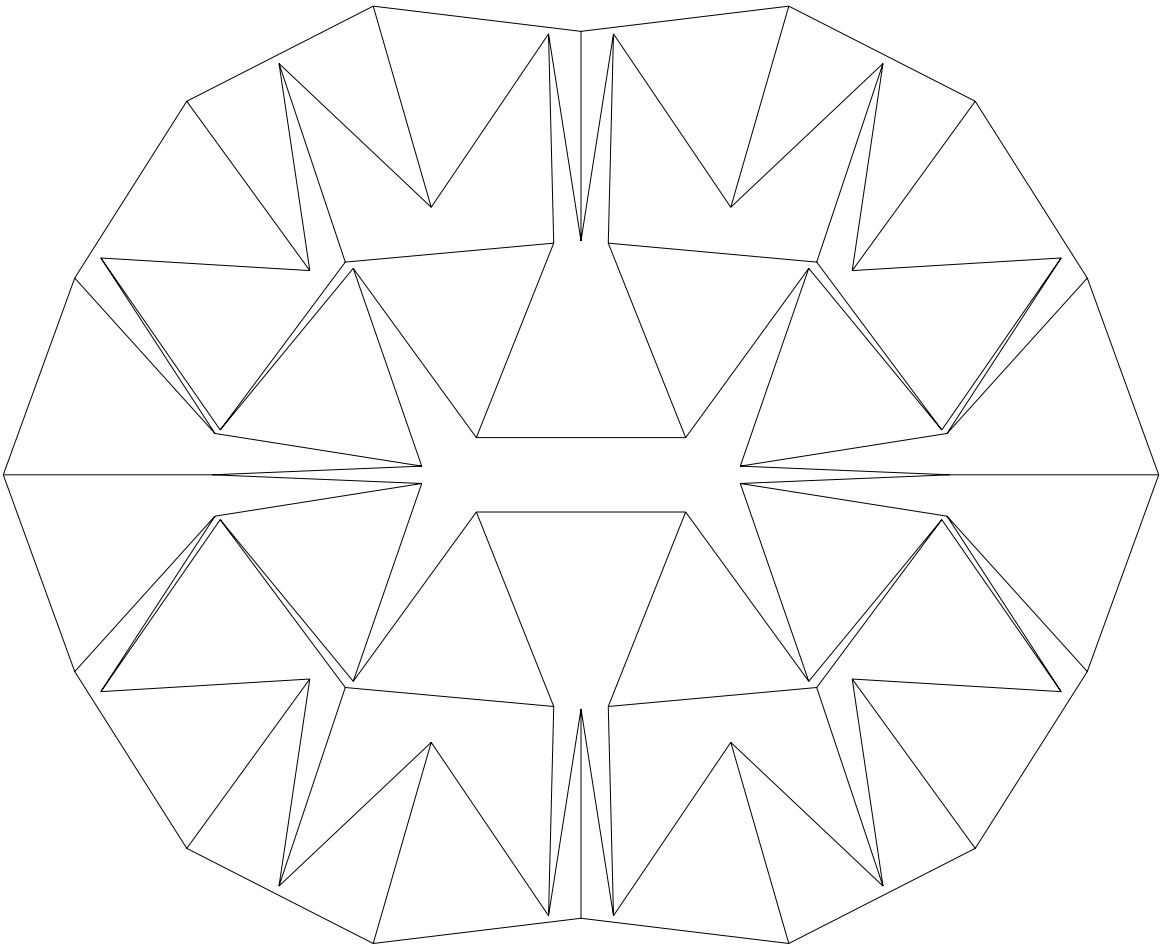

Figure 9: 68 vertices, mirror symmetry

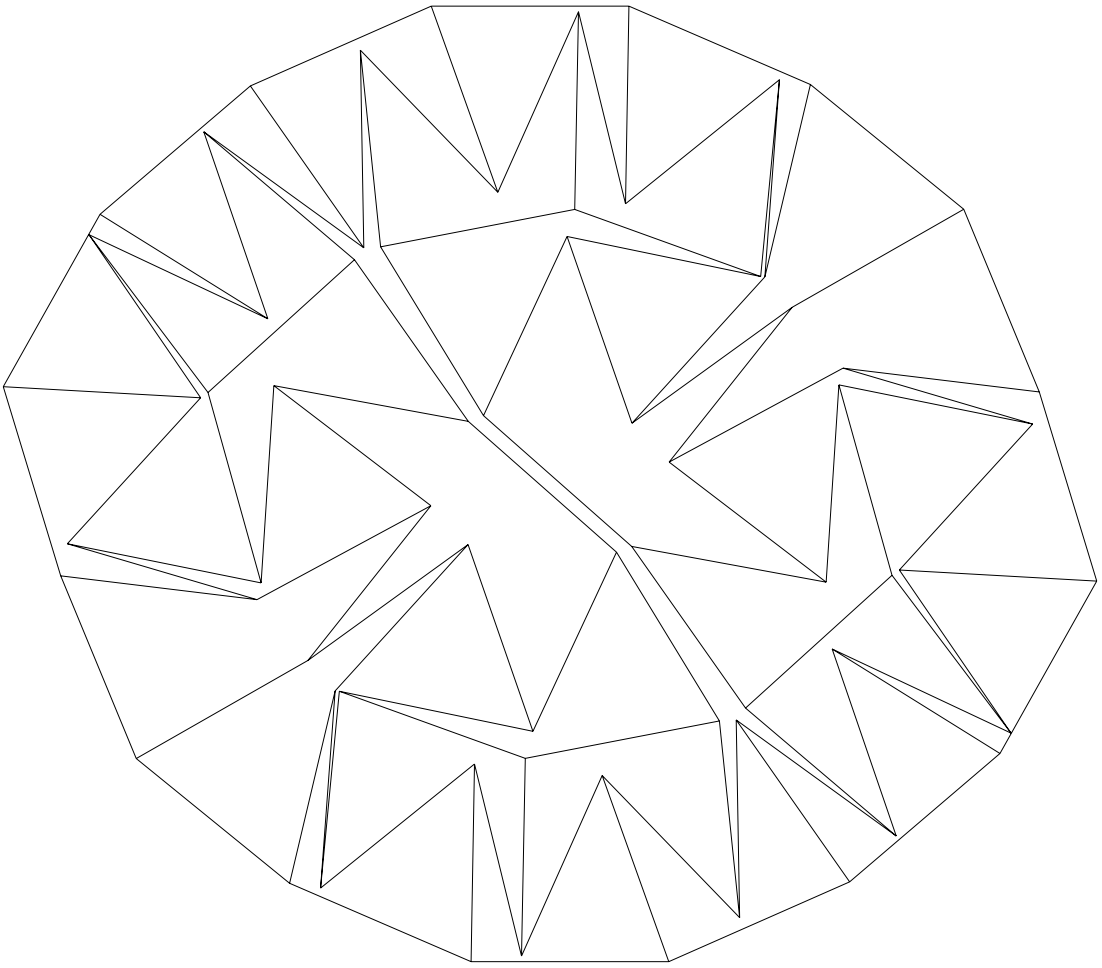

Figure 10: 68 vertices, point symmetry

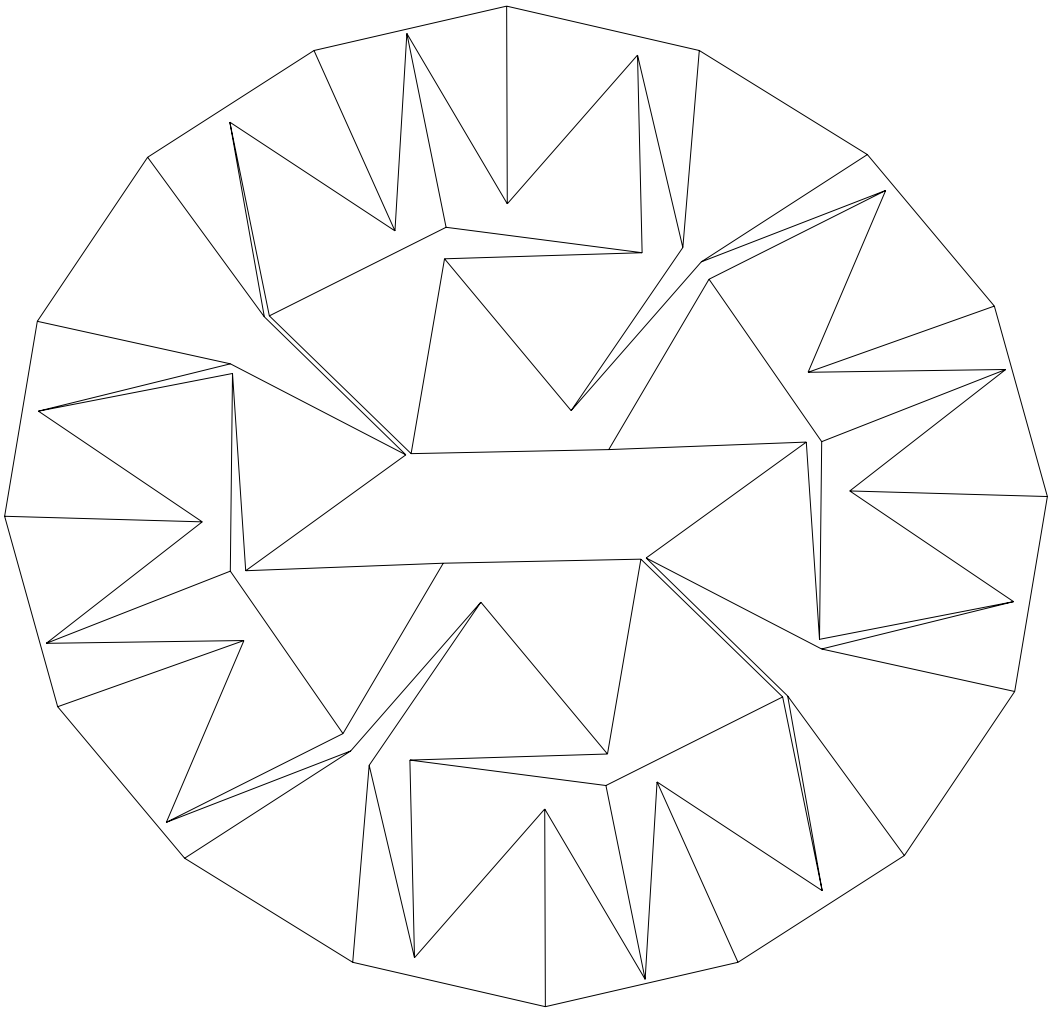

Figure 11: 68 vertices, point symmetry

3. References

[1] Sascha Kurz and Giuseppe Mazzuoccolo, *3-regular matchstick graphs with given girth*, Geombinatorics Quarterly Volume 19, Issue 4, April 2010, pp. 156–175. https://arxiv.org/abs/1401.4360

[2] Stefan Vogel, *Matchstick Graphs Calculator* (MGC), a software for the construction and calculation of unit distance graphs and matchstick graphs, (2016–2019).
http://mikematics.de/matchstick-graphs-calculator.htm

[3] Mike Winkler, Peter Dinkelacker, and Stefan Vogel, *A 3-regular matchstick graph of girth 5 consisting of 54 vertices*, March 2019.
https://arxiv.org/abs/1903.04304

[4] Mike Winkler, Peter Dinkelacker, and Stefan Vogel, *Streichholzgraphen 4-regulär und 4/n-regulär (n>4) und 2/5*, thread in a graph theory internet forum, post No.1598–No.1719, P. Dinkelacker (haribo), M. Winkler (Slash).
https://tinyurl.com/yy2jeml2